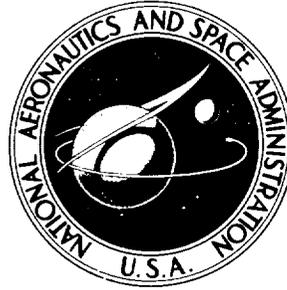


NASA TECHNICAL NOTE



NASA TN D-6702

NASA TN D-6702

COPY FILE

COMPUTER PROGRAM
FOR PRELIMINARY DESIGN ANALYSIS
OF AXIAL-FLOW TURBINES

by Arthur J. Glassman

Lewis Research Center

Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1972

1. Report No. NASA TN D-6702	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle COMPUTER PROGRAM FOR PRELIMINARY DESIGN ANALYSIS OF AXIAL-FLOW TURBINES		5. Report Date March 1972	
		6. Performing Organization Code	
7. Author(s) Arthur J. Glassman		8. Performing Organization Report No. E-6603	
		10. Work Unit No. 764-74	
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Note	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		14. Sponsoring Agency Code	
		15. Supplementary Notes	
16. Abstract <p>The program method is based on a mean-diameter flow analysis. Input design requirements include power or pressure ratio, flow, temperature, pressure, and speed. Turbine designs are generated for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse). Exit turning vanes can be included in the design. Program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, blading angles, and last-stage critical velocity ratios. The report presents the analysis method, a description of input and output with sample cases, and the program listing.</p>			
17. Key Words (Suggested by Author(s)) Turbines Axial-flow Design analysis		18. Distribution Statement Unclassified - unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 38	22. Price* \$3.00

COMPUTER PROGRAM FOR PRELIMINARY DESIGN ANALYSIS OF AXIAL-FLOW TURBINES

by Arthur J. Glassman
Lewis Research Center

SUMMARY

This report presents a computer program for the preliminary design analysis of axial-flow turbines. The computations are based on mean-diameter flow properties and do not consider any radial gradients. Given as input to the program are power or pressure ratio, mass flow rate, inlet temperature and pressure, rotative speed, inlet and exit diameters (either hub, mean, or tip), exit radius ratio or stator exit angle, turbine loss coefficient, and gas properties. Computations are then performed for any specified number of stages and for any of three types of velocity diagrams (symmetrical, zero exit swirl, or impulse). Exit turning vanes can be included in the design. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, blading angles, and last-stage critical velocity ratios.

The analysis method, a complete description of input and output, and a FORTRAN IV program listing are presented in this report. Sample cases are included to illustrate use of the program.

INTRODUCTION

The preliminary analysis of a power or propulsion system involves many repetitive calculations to determine system performance, component performance, and component geometries over a range of conditions. This must be done in order to eventually determine the best system and operating conditions. For this type of screening analysis, complete design accuracy and detail for the components are not necessary. Approximate and rapid generalized procedures rather than complex and time-consuming detailed design procedures are sufficient to yield the desired component overall geometry and performance characteristics.

This report presents a computer program for the preliminary design analysis of

axial-flow turbines. The analysis is based on mean-diameter flow properties and does not consider any radial gradients. Input design requirements include power or pressure ratio, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include inlet and exit diameters, stator angle, and number of stages. Computations are performed for any of three types of velocity diagrams (symmetrical, impulse, or zero exit swirl) by assuming the same shape diagrams for each stage. The program output includes inlet and exit annulus dimensions, exit temperature and pressure, total and static efficiencies, blading angles, and last-stage velocity ratios.

The analysis method, a complete description of input and output, and a FORTRAN IV program listing are presented in this report. Sample cases are included to illustrate use of the program.

SYMBOLS

A	stage loss parameter
A_{an}	annulus area, m^2 ; ft^2
B	exit loss parameter
C	blade loss parameter
C_A	dimensional constant, 2π rad/rev; 60 sec/min
C_B	dimensional constant, 1; 550 ft-lb/(sec)(hp)
c_p	heat capacity, joules/(kg)(K); Btu/(lb)(°R)
D	diameter, m; ft
E	squared ratio of stage-exit axial velocity to stage-average axial velocity
F	blade loss weighting factor
g	dimensional constant, 1; 32.2 ft-lbm/(sec ²)(lbf)
Δh	specific work, joules/kg; Btu/lb
i	stage number i, $i = 1, 2, \dots, n$
J	dimensional constant, 1; 778 ft-lb/Btu
K	turbine loss coefficient
M	Mach number
N	rotative speed, rad/sec; rpm
n	number of stages
P	shaft power, watts; hp

p	pressure, N/m^2 ; lb/ft^2
R	gas constant, $\text{joules}/(\text{kg})(\text{K})$; $\text{ft-lbf}/(\text{lbm})(^\circ\text{R})$
Re	Reynolds number
r	radius, m; ft
T	temperature, K; $^\circ\text{R}$
U	blade speed, m/sec; ft/sec
V	absolute gas velocity, m/sec; ft/sec
W	relative gas velocity, m/sec; ft/sec
w	mass flow rate, kg/sec; lb/sec
α	absolute-flow angle from axial direction, deg
β	relative-flow angle from axial direction, deg
γ	heat capacity ratio
η	static efficiency
η'	total efficiency
λ	speed-work parameter
μ	viscosity, $(\text{N})(\text{sec})/\text{m}^2$; $\text{lb}/(\text{sec})(\text{ft})$
ρ	density, kg/m^3 ; lb/ft^3

Subscripts:

a	first stage
cr	critical
ev	exit vane
ex	turbine exit
h	hub
i	stage i, $i = 1, 2, \dots, n$
in	turbine inlet
m	mean section
n	last stage
ro	rotor
st	stator
t	tip

u tangential component
x axial component
1 stator exit
2 rotor exit

Superscripts:

— turbine overall
' absolute total condition
'' relative total condition

METHOD OF ANALYSIS

The method is based upon an analysis of the flow at the turbine mean diameter. Radial gradients of the flow properties are not considered. Specific heat ratio is assumed constant throughout the turbine. For any given turbine, all stages, except the first, are specified to have the same shape velocity diagram. The first stage differs only in that the inlet flow is axial. The velocity diagram shape depends upon the speed-work parameter value and the specified type of velocity diagram. Three types of velocity diagram are considered: symmetrical, zero exit swirl, and impulse. These three types of velocity diagram are shown in figure 1 for three values of speed-work parameter.

Various input options dictate the exact nature of the calculation procedure. There is, however, one basic procedure that is direct and without iteration. This basic procedure will be presented and then the alternate procedures required for the various input options will be discussed. The computations can be done either in SI units or in U.S. customary units.

Basic Calculation Procedure

The required inputs for the basic procedure are shaft power P , mass flow rate w , inlet total temperature T'_{in} , inlet total pressure p'_{in} , rotative speed N , inlet mean diameter $D_{m,in}$, exit mean diameter $D_{m,ex}$, stator exit angle α_1 , gas constant R , specific heat ratio γ , viscosity μ , loss coefficient K , and squared ratio of stage-exit to stage-average axial velocities E . Also specified for each calculation are the number of stages n and the type of velocity diagram. For a multistage turbine, the input variable specified as inlet diameter is used to calculate first rotor blade speed and annulus

dimensions at the first rotor exit. Therefore, it is truly an inlet diameter only if the hub and tip diameters are assumed constant across the first stage. For a one-stage turbine, the specified exit diameter is used for the calculations, and the inlet diameter is of no significance.

The first- and last-stage blade speeds are

$$U_a = \frac{\pi N D_{m, in}}{C_A} \quad (1)$$

$$U_n = \frac{\pi N D_{m, ex}}{C_A} \quad (2)$$

For more than two stages, it is assumed that the stage blade speeds vary linearly between the first- and last-stage values. Therefore,

$$U_i = \frac{U_n - U_a}{n - 1} (i - 1) + U_a \quad (3)$$

Turbine specific work is

$$\overline{\Delta h'} = \frac{C_B}{J} \frac{P}{w} \quad (4)$$

and is equal to the sum of the specific work of the stages:

$$\overline{\Delta h'} = \sum_{i=1}^n \Delta h'_i \quad (5)$$

Expressing stage specific work in terms of stage speed-work parameter as

$$\Delta h'_i = \frac{U_i^2}{gJ\lambda} \quad (6)$$

and substituting equation (6) into equation (5) yield

$$\overline{\Delta h'} = \sum_{i=1}^n \frac{U_i^2}{gJ\lambda} \quad (7)$$

Since the velocity diagram shape is specified to be the same for all stages, the speed-work parameter is the same for all stages and is computed by rearranging equation (7) as

$$\lambda = \frac{\sum_{i=1}^n U_i^2}{gJ\overline{\Delta h'}} \quad (8)$$

The value of λ is the primary factor determining turbine efficiency.

The method used for computing turbine static efficiency is basically similar to that presented in reference 1, but has the following additional features: (1) the turbines considered in this report are not restricted to a constant mean-section diameter, (2) exit vanes to provide axial flow leaving the turbine can be included in the design, and (3) the velocity diagrams can be specified to be symmetrical. The efficiency computation method is explained fully in reference 1, and only the key equations are presented in this section.

With turbine reheat neglected, turbine static efficiency can be expressed as

$$\overline{\eta} = \frac{\overline{\Delta h'}}{\Delta h'_{id,a} + \sum_{i=2}^{n-1} \Delta h'_{id,i} + \Delta h'_{id,n}} \quad (9)$$

Dividing numerator and denominator by $\overline{\Delta h'}$ and introducing stage efficiencies yield

$$\overline{\eta} = \frac{1}{\frac{1}{\eta'_a} \frac{\Delta h'_a}{\overline{\Delta h'}} + \frac{1}{\eta'_i} \sum_{i=2}^{n-1} \frac{\Delta h'_i}{\overline{\Delta h'}} + \frac{1}{\eta'_n} \frac{\Delta h'_n}{\overline{\Delta h'}}} \quad (10)$$

Dividing equation (6) by equation (7) shows that

$$\frac{\Delta h'_1}{\overline{\Delta h'}} = \frac{U_1^2}{\sum_{i=1}^n U_i^2} \quad (11)$$

Substituting equation (11) into equation (10) and recognizing that

$$\sum_{i=2}^{n-1} \left(\frac{U_i^2}{\sum_{i=1}^n U_i^2} \right) = 1 - \frac{U_a^2}{\sum_{i=1}^n U_i^2} - \frac{U_n^2}{\sum_{i=1}^n U_i^2} \quad (12)$$

finally yield

$$\bar{\eta} = \frac{1}{\frac{1}{\eta'_a} \frac{U_a^2}{\sum_{i=1}^n U_i^2} + \frac{1}{\eta'_i} \left(1 - \frac{U_a^2}{\sum_{i=1}^n U_i^2} - \frac{U_n^2}{\sum_{i=1}^n U_i^2} \right) + \frac{1}{\eta_n} \frac{U_n^2}{\sum_{i=1}^n U_i^2}} \quad (13)$$

The stage-total and last-stage-static efficiencies are

$$\eta' = \frac{\lambda}{\lambda + \frac{A}{2}} \quad (14)$$

and

$$\eta_n = \frac{\lambda}{\lambda + \frac{1}{2}(A + B)} \quad (15)$$

The stage loss parameter A is expressed as

$$A = \frac{KRe^{-0.2}}{\cot \alpha_1} (F_{st}C_{st} + F_{ro}C_{ro} + C_{ev}) \quad (16)$$

The constant of proportionality K , called the turbine loss coefficient in this report, must be determined empirically. On the basis of comparisons of predicted with experimental efficiencies, a value of $K = 0.4$ was selected in reference 1. For large turbines of recent airbreathing engines, a value of $K = 0.35$ seems better. The Reynolds number used in this calculation is defined as

$$Re = \frac{2w}{\mu D_{m,in}} \quad (17)$$

Some of the terms within the parentheses in equation (16) are the same for all cases, while others depend on stage location, velocity diagram type, and use of exit vanes. The rotor weighting factor F_{ro} and rotor loss parameter C_{ro} are the same for all cases:

$$F_{ro} = 2 \quad (18)$$

$$C_{ro} = 2 \cot^2 \alpha_1 \left(\frac{V_{u,1}}{\Delta V_u} \right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \lambda \right)^2 + \left(\frac{V_{u,2}}{\Delta V_u} - \lambda \right)^2 \quad (19)$$

For all stages other than last stages and for last stages where exit vanes are not used, the exit vane loss parameter is

$$C_{ev} = 0 \quad (20)$$

For last stages of turbines having exit vanes,

$$C_{ev} = 2 \cot^2 \alpha_1 \left(\frac{V_{u,1}}{\Delta V_u} \right)^2 + \left(\frac{V_{u,2}}{\Delta V_u} \right)^2 \quad (21)$$

Axial inlet flow is assumed for all first-stage stators, for which the stator loss parameter is expressed

$$C_{st} = \left(1 + 2 \cot^2 \alpha_1\right) \left(\frac{V_{u,1}}{\Delta V_u}\right)^2 \quad (22)$$

For all stators other than first-stage stators,

$$C_{st} = \left(1 + 2 \cot^2 \alpha_1\right) \left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,2}}{\Delta V_u}\right)^2 \quad (23)$$

The stator weighting factor F_{st} also depends on whether or not the stator is a first-stage stator and further depends on the type of velocity diagram. The inlet and exit swirl parameters $V_{u,1}/\Delta V_u$ and $V_{u,2}/\Delta V_u$ of equations (19) to (23) also depend on the type of velocity diagram. The following table presents the relations for evaluating the stator weighting factor and the swirl parameters:

Stage	Velocity diagram type	Stator weighting factor, F_{st}	Inlet swirl parameter, $V_{u,1}/\Delta V_u$	Exit swirl parameter, $V_{u,2}/\Delta V_u$
First	Symmetrical	1	$\frac{\lambda + 1}{2}$	$\frac{\lambda - 1}{2}$
	Zero exit swirl	1	1	0
	Impulse with $\lambda \leq 0.5$	1	$\lambda + \frac{1}{2}$	$\lambda - \frac{1}{2}$
	Impulse with $\lambda \geq 0.5$	1	$\lambda + \frac{1}{2}$	$\lambda - \frac{1}{2}$
Intermediate and last	Symmetrical	$2 - \lambda$	$\frac{\lambda + 1}{2}$	$\frac{\lambda - 1}{2}$
	Zero exit swirl	1	1	0
	Impulse with $\lambda \leq 0.5$	$2(1 - \lambda)$	$\lambda + \frac{1}{2}$	$\lambda - \frac{1}{2}$
	Impulse with $\lambda \geq 0.5$	1	$\lambda + \frac{1}{2}$	$\lambda - \frac{1}{2}$

The exit loss parameter B of equation (15) is expressed as

$$B = E \cot^2 \alpha_1 \left(\frac{V_{u,1}}{\Delta V_u} \right)^2 + \left(\frac{V_{u,2}}{\Delta V_u} \right)_{\text{ex}}^2 \quad (24)$$

where

$$\left(\frac{V_{u,2}}{\Delta V_u} \right)_{\text{ex}} = \frac{V_{u,2}}{\Delta V_u} \quad (25)$$

with no exit vanes and

$$\left(\frac{V_{u,2}}{\Delta V_u} \right)_{\text{ex}} = 0 \quad (26)$$

with exit vanes.

The turbine exit velocities and state conditions are computed as follows:

$$\Delta V_{u,n} = \frac{U_n}{\lambda} \quad (27)$$

$$V_{u,1,n} = \frac{V_{u,1}}{\Delta V_u} \Delta V_{u,n} \quad (28)$$

$$V_{u,2,n} = \frac{V_{u,2}}{\Delta V_u} \Delta V_{u,n} \quad (29)$$

$$V_{x,n} = V_{u,1,n} \cot \alpha_1 \quad (30)$$

$$V_{x,\text{ex}} = \sqrt{E} V_{x,n} \quad (31)$$

$$V_{u,\text{ex}} = \begin{cases} V_{u,2,n} & \text{no exit vanes} \\ 0 & \text{exit vanes} \end{cases} \quad (32)$$

$$V_{ex} = \sqrt{V_{x,ex}^2 + V_{u,ex}^2} \quad (33)$$

$$c_p = \frac{\gamma}{\gamma - 1} \frac{R}{J} \quad (34)$$

$$p_{ex} = p'_{in} \left(1 - \frac{\overline{\Delta h'}}{c_p T'_{in} \bar{\eta}} \right)^{\gamma/(\gamma-1)} \quad (35)$$

$$T'_{ex} = T'_{in} - \frac{\overline{\Delta h'}}{c_p} \quad (36)$$

$$T_{ex} = T'_{ex} - \frac{V_{ex}^2}{2gJc_p} \quad (37)$$

$$p'_{ex} = p_{ex} \left(\frac{T'_{ex}}{T_{ex}} \right)^{\gamma/(\gamma-1)} \quad (38)$$

In order that the turbine total and static efficiencies be consistent with the computed exit velocity, the total efficiency is computed as

$$\bar{\eta}' = \frac{\overline{\Delta h'}}{c_p T'_{in} \left[1 - \left(\frac{p'_{ex}}{p'_{in}} \right)^{(\gamma-1)/\gamma} \right]} \quad (39)$$

Exit annulus area, radius ratio, and hub and tip diameters are obtained as follows:

$$\rho_{ex} = \frac{p_{ex}}{RT_{ex}} \quad (40)$$

$$A_{an,ex} = \frac{w}{\rho_{ex} V_{x,ex}} \quad (41)$$

$$\left(\frac{r_h}{r_t}\right)_{\text{ex}} = \frac{1 - \frac{A_{\text{an,ex}}}{\pi D_{\text{m,ex}}^2}}{1 + \frac{A_{\text{an,ex}}}{\pi D_{\text{m,ex}}^2}} \quad (42)$$

$$D_{\text{t,ex}} = \frac{2D_{\text{m,ex}}}{1 + \left(\frac{r_h}{r_t}\right)_{\text{ex}}} \quad (43)$$

$$D_{\text{h,ex}} = D_{\text{t,ex}} \left(\frac{r_h}{r_t}\right)_{\text{ex}} \quad (44)$$

Absolute and relative flow angles, which are the same for each stage, are computed from the last-stage velocities:

$$\alpha_2 = \tan^{-1} \frac{V_{\text{u,2,n}}}{V_{\text{x,n}}} \quad (45)$$

$$W_{\text{u,1,n}} = V_{\text{u,1,n}} - U_{\text{n}} \quad (46)$$

$$W_{\text{u,2,n}} = V_{\text{u,2,n}} - U_{\text{n}} \quad (47)$$

$$\beta_1 = \tan^{-1} \frac{W_{\text{u,1,n}}}{V_{\text{x,n}}} \quad (48)$$

$$\beta_2 = \tan^{-1} \frac{W_{\text{u,2,n}}}{V_{\text{x,n}}} \quad (49)$$

Critical velocity ratios are computed for the last stage, where temperatures are lowest and the velocity ratios are most severe:

$$V_{2,n} = \sqrt{V_{\text{u,2,n}}^2 + V_{\text{x,ex}}^2} \quad (50)$$

$$W_{2,n} = \sqrt{W_{u,2,n}^2 + V_{x,ex}^2} \quad (51)$$

$$W_{1,n} = \sqrt{W_{u,1,n}^2 + V_{x,n}^2} \quad (52)$$

$$V_{1,n} = \sqrt{V_{u,1,n}^2 + V_{x,n}^2} \quad (53)$$

$$T_{2,n}'' = T_{1,n}' = T_{ex}' - \frac{V_{2,n}^2 - W_{2,n}^2}{2gJc_p} \quad (54)$$

$$T_{1,n}' = T_{1,n}'' - \frac{W_{1,n}^2 - V_{1,n}^2}{2gJc_p} \quad (55)$$

$$\left(\frac{V_2}{V_{cr,2}}\right)_n = \frac{V_{2,n}}{\sqrt{2 \frac{\gamma}{\gamma+1} gRT_{ex}'}} \quad (56)$$

$$\left(\frac{W_2}{W_{cr,2}}\right)_n = \frac{W_{2,n}}{\sqrt{2 \frac{\gamma}{\gamma+1} gRT_{2,n}''}} \quad (57)$$

$$\left(\frac{W_1}{W_{cr,1}}\right)_n = \frac{W_{1,n}}{\sqrt{2 \frac{\gamma}{\gamma+1} gRT_{1,n}'}} \quad (58)$$

$$\left(\frac{V_1}{V_{cr,2}}\right)_n = \frac{V_{1,n}}{\sqrt{2 \frac{\gamma}{\gamma+1} gRT_{1,n}'}} \quad (59)$$

$$M_{x,ex} = \frac{V_{x,ex}}{\sqrt{\gamma gRT_{ex}'}} \quad (60)$$

In order to establish the flow annulus geometry near the turbine inlet, a flow analysis is made at the first-stage exit as follows:

$$\Delta h'_a = \frac{U_a^2}{gJ\lambda} \quad (61)$$

$$T'_{2,a} = T'_{in} - \frac{\Delta h'_a}{c_p} \quad (62)$$

$$p'_{2,a} = p'_{in} \left(1 - \frac{\Delta h'_a}{c_p T'_{in} \eta'_a} \right)^{\gamma/(\gamma-1)} \quad (63)$$

$$\Delta V_{u,a} = \frac{U_a}{\lambda} \quad (64)$$

$$V_{u,2,a} = \frac{V_{u,2}}{\Delta V_u} \Delta V_{u,a} \quad (65)$$

$$V_{x,2,a} = \sqrt{E} \frac{V_{u,1}}{\Delta V_u} \Delta V_{u,a} \cot \alpha_1 \quad (66)$$

$$V_{2,a} = \sqrt{V_{u,2,a}^2 + V_{x,2,a}^2} \quad (67)$$

$$T_{2,a} = T'_{2,a} - \frac{V_{2,a}^2}{2gJc_p} \quad (68)$$

$$p_{2,a} = p'_{2,a} \left(\frac{T_{2,a}}{T'_{2,a}} \right)^{\gamma/(\gamma-1)} \quad (69)$$

$$\rho_{2,a} = \frac{p_{2,a}}{RT_{2,a}} \quad (70)$$

$$A_{an,2,a} = \frac{w}{\rho_{2,a} V_{x,2,a}} \quad (71)$$

$$\left(\frac{r_h}{r_t}\right)_{2,a} = \frac{1 - \frac{A_{an,2,a}}{\pi D_{m,in}^2}}{1 + \frac{A_{an,2,a}}{\pi D_{m,in}^2}} \quad (72)$$

$$D_{t,2,a} = \frac{2D_{m,in}}{1 + \left(\frac{r_h}{r_t}\right)_{2,a}} \quad (73)$$

$$D_{h,2,a} = D_{t,2,a} \left(\frac{r_h}{r_t}\right)_{2,a} \quad (74)$$

When a constant annulus is assumed for the first stage, the first-stage exit dimensions become the turbine inlet dimensions.

Alternative Calculation Procedure

The basic calculation procedure described in the previous section requires as inputs the inlet and exit mean diameters, stator exit angle, and shaft power. Alternatively, the hub or tip diameters could be specified as input and the mean diameters computed, the exit radius ratio could be specified as input and the stator exit angle computed, and the turbine pressure ratio could be specified as input and the shaft power computed. These alternative input options require iterative calculation procedures such as described in this section.

With hub or tip diameters rather than mean diameters specified at the inlet and exit, it is necessary to assume initial values for the inlet and exit radius ratios. Initial values for inlet and exit mean diameters are then obtained as

$$D_m = \frac{\left(1 + \frac{r_h}{r_t}\right) D_h}{2 \frac{r_h}{r_t}} \quad (75)$$

when hub diameter is input and

$$D_m = \frac{\left(1 + \frac{r_h}{r_t}\right) D_t}{2} \quad (76)$$

when tip diameter is input. The computation then proceeds from equation (1) through equation (42) and the computed exit radius ratio is compared with the assumed value. If they are not the same (within a given tolerance), then the computed value of exit radius ratio is used to calculate a new value for exit mean diameter (from eq. (75) or (76)) and the computation procedure is repeated until convergence is obtained. Then, computation proceeds through equation (72) and the computed inlet radius ratio is compared with the assumed value. If they are not the same, the computed value of inlet radius ratio is used to calculate a new value for inlet mean diameter, and the computation procedure is repeated from equation (1). This entire procedure is repeated until both inlet and exit radius ratios in the same calculation pass converge to previous values.

With exit radius ratio rather than stator exit angle specified, a value of stator exit angle is assumed for the evaluation of equation (16). The computation proceeds through equation (40). Equation (42) is then used to compute the exit annulus area from the input value of radius ratio, and the exit axial velocity is then obtained from equation (41). The density used in equation (41), however, is not consistent with the exit area, and equations (41), (33), (37), and (40) must be iterated until convergence is obtained. Then, the stator exit angle is computed as

$$\alpha_1 = \tan^{-1} \frac{V_{u,1,n}}{V_{x,n}} \quad (77)$$

and compared with the assumed value. If they are not the same, the computed value of stator exit angle is used for the evaluation of equation (16), and the computation procedure just given is repeated until two consecutive values of stator exit angle are the same. The remainder of the computation is then completed.

With the turbine inlet-total- to exit-static-pressure ratio rather than shaft power specified as input, an initial value of turbine static efficiency is assumed. Turbine work is then computed from

$$\overline{\Delta h'} = \bar{\eta}_c p_{in}' T_{in}' \left[1 - \left(\frac{p_{ex}}{p_{in}'} \right)^{(\gamma-1)/\gamma} \right] \quad (78)$$

instead of from equation (4). The computation then proceeds through equation (26) in order to compute a static efficiency from equation (16). If the computed value is not the same as the assumed value, a new value of static efficiency is assumed, and the computation is repeated until two successive values are the same. The remainder of the computation is then completed.

DESCRIPTION OF INPUT AND OUTPUT

This section presents a detailed description of the program input, normal output, and error messages. Included in the input and output sections are several example cases illustrating the use of the program and the various options.

Input

The program input, a sample of which is presented in table I, consists of a title card and the required physical data and option indicators in NAMELIST form. The title, which is printed as a heading on the output listing, can contain up to 77 characters located anywhere in columns 2 to 78 on the title card. A title card, even if it is left blank, must be the first card of the data package. Additional title cards can be used to identify different cases being run in the same data package. This is done by placing a title card in front of the data record for the particular case and using the option indicator ITIT as subsequently described.

The physical data and option indicators are input in data records having the NAMELIST name INPUT. The variables and indicators that compose INPUT and the proper units are as follows. These must be inputted for all cases except where otherwise indicated. Either the SI units or the U.S. customary units shown after them may be used.

PTIN	inlet total pressure, N/cm^2 ; $\text{lb}/\text{in.}^2$
TTIN	inlet total temperature, K; $^{\circ}\text{R}$
MU	gas viscosity, $(\text{N})(\text{sec})/\text{m}^2$; $\text{lb}/(\text{sec})(\text{ft})$
R	gas constant, $\text{joules}/(\text{kg})(\text{K})$; $\text{ft}\text{-lb}/(\text{lbm})(^{\circ}\text{R})$
GAM	heat capacity ratio
DIN	inlet diameter - hub or mean or tip value as specified by the indicator IDIAM, cm; in.
DEX	exit diameter - hub or mean or tip value as specified by the indicator IDIAM, cm; in.
RREX	exit radius ratio; RREX may be omitted in the case where both IDIAM = 2 and IALPH = 0; RREX is used as first trial value when IALPH = 0 and IDIAM = 1 or 3
RPM	rotative speed, rad/sec ; rpm
POW	shaft power - omit when IPR = 1, kW; hp
W	mass flow rate, kg/sec ; lb/sec
ALPHA	stator exit angle from axial direction; ALPHA is used as first trial value when IALPH = 1, deg
KLOSS	turbine loss coefficient; a value in the range of 0.35 to 0.40 is usually applicable
NMIN	minimum number of stages for which the calculations are performed
NMAX	maximum number of stages for which the calculations are performed; results are obtained for all stage numbers between NMIN and NMAX
E	squared ratio of stage-exit to stage-average axial velocities
PRTS	turbine inlet-total- to exit-static-pressure ratio; omit when IPR = 0
IALPH	indicates whether stator exit angle or turbine exit radius ratio is specified: IALPH = 0 - turbine is designed for specified ALPHA IALPH = 1 - turbine is designed for specified RREX
IDIAM	indicates whether inputted diameters are hub, mean, or tip values: IDIAM = 1 - inputted diameters are hub values IDIAM = 2 - inputted diameters are mean values IDIAM = 3 - inputted diameters are tip values

- IVD indicates type of velocity diagram used:
 IVD = 1 - symmetrical diagrams
 IVD = 2 - zero exit swirl diagrams
 IVD = 3 - impulse diagrams
 IVD = 4 - zero exit swirl diagrams if $\lambda \geq 0.5$ and impulse diagrams if $\lambda \leq 0.5$
- ITIT indicates use of title cards in addition to that required as first card of data package:
 ITIT = 1 - title card precedes next data set; must be inputted for each additional title card because ITIT is automatically restored to zero after each title card is read
- IEV indicates use of exit vanes:
 IEV = 0 - no exit vanes
 IEV = 1 - exit vanes are used to turn turbine exit flow to axial direction
- IPR indicates whether shaft power or pressure ratio is specified:
 IPR = 0 - shaft power is inputted
 IPR = 1 - turbine inlet-total- to exit-static-pressure ratio is inputted
- IU indicates type of units used for input and output:
 IU = 1 - SI units
 IU = 2 - U.S. customary units

Each line of the input form shown in table I represents one data card. The first card is the mandatory title card, which can contain any desired message. The next three cards are the first data set, which contains all required inputs. This first case represents computation in accordance with the basic calculation procedure described previously. Data inputted for subsequent cases need only include those values that differ from previous case data. The fifth card is the second data set and represents the option where hub diameter is input. Also, the second case data specify that a title card, which is the sixth data card, precedes the third case data. Cards 7 to 10 represent four additional cases illustrating use of different input options. The output corresponding to this sample input is described in the following section.

Output

The program output consists of title headings, the input variables, and computed results. This section presents normal output. Error message output is described in the next section.

Table II presents the output that corresponds to the sample input shown in table I. The top line of output is a program identification title that is automatically printed. The second line is the title card message. The next four lines are the input variables and their associated values for the first data set. The input variable names are spelled out. The units for the input variable values are as described in the Input section. The zeros printed under EXIT RADIUS RATIO and T-S PRESSURE RATIO indicate that these are computed for this case and not specified by the input. The fact that the input diameters are mean diameters is indicated by the MN in the variable name. Hub and tip diameters would be indicated by HB and TP, respectively. These four lines of output are printed for each new data set.

The next two groups of eight lines each are the computation results for a one-stage turbine and a two-stage turbine, each satisfying the input requirements. Only one- and two-stage designs were specified by the input. The output parameters are spelled out and are self-explanatory. On the first line of each group are the number of stages, the stage speed-work parameter, and the diagram type, which is symmetrical for this first case. The remainder of the output includes exit and inlet tip and hub diameters in the first column, exit total and static temperatures and pressures in the second column, total and static efficiencies and velocity diagram angles in the third column, and first- and last-stage blade speeds and last-stage absolute velocity components in the last column. The last line of each output group presents the last-stage absolute and relative critical velocity ratios.

After the computations for each input case are completed, the input data for the next case are printed. The second case presented here is that where the specified diameters are hub values. The third input case in table I is preceded by an additional title card. This causes the next output to begin at the top of a new page with the program identification title and the title card message. The third to sixth cases are computed for two stages only.

The third case is for a specified exit radius ratio rather than for a specified stator exit angle. This is indicated by a zero appearing under STATOR EX ANG in the row of input variables. Exit vanes are included in this case, as indicated by WITH EXIT VANES printed after number of stages on the first line of result output. The fourth and fifth cases, as indicated by the top line of the fourth column of result output, are for zero exit swirl and impulse diagrams, respectively. Turbine pressure ratio rather than shaft power is specified in the sixth case. For this case, the inputted pressure ratio was chosen to be the same as the computed value obtained for the two-stage design of the second case. It is seen that, as should be expected, the program converges to identical solutions for both cases. An extra line of output consisting of shaft power and specific work is printed for the case where pressure ratio is specified as input.

Error Messages

The program contains five output messages indicating the nonexistence of a solution satisfying the specified input requirements. These messages are presented in this section, and their causes are discussed.

(1) **INSUFFICIENT ENERGY** - This message is caused by the computed turbine exit total temperature being less than zero. It indicates that the turbine specific work requirement is greater than the energy available in the gas. Therefore, either the specified shaft power must be decreased or the specified flow must be increased.

(2) **INSUFFICIENT IDEAL ENERGY** - This message is caused by the computed ideal energy being more than that available from an infinite expansion of the gas. It indicates that the computed static efficiency is too low to yield a valid solution. Corrective action includes decreasing power, increasing flow, using more stages, or perhaps using a different velocity diagram.

(3) **NEGATIVE TEX** - This message is caused by the computed turbine exit static temperature being less than zero. It indicates a low value of turbine exit total temperature and/or a high value of turbine exit velocity. Corrective action could be the same as for message (2) or decreasing a too high value of stage exit to average axial kinetic energy ratio.

(4) **INSUFFICIENT EXIT AREA** - This message is caused by the computed exit area being larger than that available in the turbine. Such a situation can be remedied in many ways, including increasing exit diameter, decreasing stator exit angle, increasing inlet pressure, decreasing mass flow, and increasing stage exit to average axial kinetic energy ratio.

(5) **INSUFFICIENT INLET AREA** - This message is caused by the computed inlet area being larger than that available in the turbine. Corrective measures are the same as for message (4).

PROGRAM DESCRIPTION

The computer program is called TURBAN. All computations are performed in one main program written in IBM 7090/7094 FORTRAN IV language. The program variables are defined in this section and the program listing is presented.

Program Variables

A	factor in eq. (16), $KRe^{-0.2}/\cot \alpha_1$
AA	factor in eq. (42), $A_{an,ex}/\pi D_{m,ex}^2$

AA1 factor in eq. (72), $A_{an,2,a}/\pi D_{m,in}^2$
AEX turbine exit annulus area
AI stage loss parameter for intermediate stage and last stage having no exit vanes
AIL stage loss parameter for last stage having exit vanes
ALP stator exit angle value printed in input section of output
ALPH previous value of stator exit angle
ALPHA input value of stator exit angle
ALPHA1 output value of stator exit angle
ALPHA2 output value of stage exit angle
ALPH1 stator exit angle
ALPH2 stage exit angle
ASEX speed of sound at turbine exit
A1 stage loss parameter for first stage or single stage having no exit vanes
A1L stage loss parameter for single stage having exit vanes
A2¹ first-stage-exit annulus area
B turbine exit loss parameter with no exit vanes
BETA1 output value of rotor inlet angle
BETA2 output value of rotor exit angle
BET1 rotor inlet angle
BET2 rotor exit angle
BL turbine exit loss parameter with exit vanes
CCN dimensional constant
CCP dimensional constant
CI blade row loss parameter for intermediate stage stator
CL blade row loss parameter for exit vanes
CONV tolerance for radius ratio convergence
COT cotangent of stator exit angle
CP heat capacity
C1 blade row loss parameter for first-stage stator

D	blade loss parameter for rotor
DELHT	turbine specific work
DELH1	first-stage specific work
DEX	input value of turbine exit diameter
DH	data statement word HB for output use
DHEX	turbine exit hub diameter
DHID	turbine ideal work based on inlet-total- to exit-static-pressure ratio
DHTID	turbine ideal work based on inlet-total- to exit-total-pressure ratio
DH1	first-stage-exit hub diameter
DIN	input value of first-stage-exit diameter
DM	data statement word MN for output use
DMEX	turbine exit mean diameter
DMIN	first-stage-exit mean diameter
DN	turbine exit mean diameter
DT	data statement word TP for output use
DTEX	turbine exit tip diameter
DT1	first-stage-exit tip diameter
DVUN	change in swirl velocity across last-stage rotor
DVU1	change in swirl velocity across first-stage rotor
DX	output word set equal to DH, DM, or DT as appropriate
D1	first-stage-exit mean diameter
E	ratio of stage-exit to stage-average axial kinetic energies
ES	turbine static efficiency
ESA	previous value of turbine static efficiency
ESI	static efficiency of last stage with no exit vanes
ESIL	static efficiency of last stage with exit vanes
ES1	static efficiency of single stage with no exit vanes
ES1L	static efficiency of single stage with exit vanes
ET	turbine total efficiency
ETI	total efficiency of intermediate stage or last stage with no exit vanes

ETIL	total efficiency of last stage with exit vanes
ET1	total efficiency of first stage or single stage with no exit vanes
ET1L	total efficiency of single stage with exit vanes
FLS	stator weighting factor
FLSL	exit vane weighting factor
G	dimensional constant
GAM	heat capacity ratio
IALPH	option indicator - see Input section
IDIAM	option indicator - see Input section
IEV	option indicator - see Input section
IPR	option indicator - see Input section
ITIT	option indicator - see Input section
IU	option indicator - see Input section
IVD	option indicator - see Input section
J	dimensional constant
KLOSS	turbine loss coefficient
LAM	stage speed-work parameter
MU	gas viscosity
MXEX	turbine exit axial Mach number
N	number of stages
NMAX	maximum number of stages
NMIN	minimum number of stages
NN	number of stages
PEX	turbine exit static pressure
PI	π
POW	shaft power
PRS	computed value of inlet-total- to exit-static-pressure ratio
PRT	computed value of inlet-total- to exit-total-pressure ratio
PRTS	input value of inlet-total- to exit-static-pressure ratio
PTEX	turbine exit total pressure

PTIN	turbine inlet total pressure
PT21	first-stage-exit total pressure
P21	first-stage-exit static pressure
Q	ratio of rotor inlet swirl velocity to change in swirl velocity
Q2	ratio of rotor exit swirl velocity to change in swirl velocity
R	gas constant
RE	Reynolds number
REX	computed value of turbine exit radius ratio
REX1	previous value of turbine exit radius ratio
RHOEX	turbine exit gas density
RHOEX1	previous value of turbine exit gas density
RHO21	first-stage-exit gas density
RPM	rotative speed
RREX	input value of turbine exit radius ratio
R1	first-stage-exit radius ratio
R11	previous value of first-stage-exit radius ratio
ST	output word set equal to ST1 or ST2 as appropriate
ST1	blank data statement words for output use
ST2	data statement words WITH EXIT VANES for output use
SUMUSQ	sum of squares of stage blade speeds
TEX	turbine exit static temperature
TITLE	input/output array for title card message
TTEX	turbine exit total temperature
TTIN	turbine inlet total temperature
TTRN	last-stage-rotor inlet and exit relative total temperature
TT1N	last-stage-stator exit absolute total temperature
TT21	first-stage-exit absolute total temperature
T21	first-stage-exit static temperature
U	rotor mean blade speed
UISQ	rotor mean blade speed squared

VCR1N	last-stage-stator-exit absolute critical velocity
VCR2N	last-stage-exit absolute critical velocity
VD1	data statement word SYMMET for output use
VD2	data statement word RICAL for output use
VD3	data statement word (blank) for output use
VD4	data statement word ZERO E for output use
VD5	data statement word XIT SW for output use
VD6	data statement word HIRL for output use
VD7	data statement word IMPULS for output use
VD8	data statement word E for output use
VU1N	last-stage-rotor inlet swirl velocity
VU11	first-stage-rotor inlet swirl velocity
VU2N	last-stage-rotor exit swirl velocity
VU21	first-stage-exit swirl velocity
VXN	turbine exit axial velocity
VXND	last-stage average axial velocity
VX1	first-stage exit axial velocity
VX11	output word set equal to VD1, VD4, or VD7 as appropriate
VX2	output word set equal to VD2, VD5, or VD8 as appropriate
VX3	output word set equal to VD3 or VD6 as appropriate
V1N	last-stage-stator-exit absolute velocity
V1OVCR	last-stage-stator-exit critical velocity ratio
V2N	turbine exit absolute velocity
V2NR	last-stage-rotor-exit absolute velocity
V2OVCR	last-stage-rotor-exit absolute critical velocity ratio
V21	first-stage-exit absolute velocity
W	mass flow rate
WC	factor in eqs. (56) to (59), $[2\gamma gR/(\gamma + 1)]^{1/2}$
WCRN	last-stage-rotor-inlet and exit relative critical velocity
WU1N	tangential component of last-stage-rotor-inlet relative velocity

WU2N tangential component of last-stage-rotor-exit relative velocity
 W1N last-stage-rotor-inlet relative velocity
 W1OWCR last-stage-rotor-inlet relative critical velocity ratio
 W2N last-stage-rotor-exit relative velocity
 W2OWCR last-stage-rotor-exit relative critical velocity ratio
 X function of γ , $\gamma/(\gamma - 1)$
 XREX exit radius ratio value printed in input section of output
 ZZ factor in eq. (35), $\overline{\Delta h'} / c_p T_{in}' \overline{\eta}$

Program Listing

\$IBFTC TURBAN DECK

```

C
C THIS PROGRAM PERFORMS TURBINE GEOMETRY AND EFFICIENCY CALCS ON A
C MEAN SECTION BASIS ASSUMING SAME SHAPE DIAGRAMS FOR EACH STAGE
C (EXCEPT FIRST, WHICH HAS AXIAL INLET FLOW) AND IF
C IVD=1 - SYMMETRICAL DIAGRAMS
C IVD=2 - ZERO EXIT SWIRL DIAGRAMS
C IVD=3 - IMPULSE DIAGRAMS
C IVD=4 - ZES FOR LAM.GE.0.5 AND IMP FOR LAM.LE.0.5
C IALPH=0 - EXIT RADIUS RATIO IS COMPUTED FOR INPUT VALUE OF ALPHA
C - INPUT RREX IS FIRST TRIAL VALUE IF IDIAM=1 OR 3
C IALPH=1 - ALPHA IS COMPUTED FOR INPUT VALUE OF EXIT RADIUS RATIO
C - INPUT ALPHA IS FIRST TRIAL VALUE
C DIAMETERS ARE INPUT AT INLET AND EXIT, AND BLADE SPEED VARIES LIN.
C IDIAM=1 - INPUT DIAMETERS ARE HUB VALUES
C IDIAM=2 - INPUT DIAMETERS ARE MEAN VALUES
C IDIAM=3 - INPUT DIAMETERS ARE TIP VALUES
C IEV=0 - NO EXIT VANES
C IEV=1 - EXIT VANES TO TURN FLOW TO AXIAL DIRECTION
C IPR=0 - POWER IS INPUT AND PRESSURE RATIO IS COMPUTED
C IPR=1 - PRESSURE RATIO(T-S) IS INPUT AND POWER IS COMPUTED
C ITIT=1 - TITLE CARD PRECEDES NEXT DATA SET
C IU=1 - SI UNITS ARE USED FOR INPUT AND OUTPUT
C IU=2 - U.S. CUSTOMARY UNITS ARE USED FOR INPUT AND OUTPUT
C
REAL LAM,NN,J,MXEX,KLOSS,MU
DIMENSION U(99),TITLE(13),ST1(3),ST2(3),ST(3)
NAMELIST/INPUT/PTIN,TTIN,MU,R,GAM,DIN,DEX,RREX,RPM,POW,W,ALPHA,
1KLOSS,IALPH,NMIN,NMAX,IDIAM,E,IVD,ITIT,IEV,IPR,PRTS,IU
DATA DH,DM,DT/2HHB,2HMN,ZHTP/
DATA VD1,VD2,VD3,VD4,VD5,VD6,VD7,VD8/6HSYMMET,5HRICAL,1H ,6HZERO E
1,6HXIT SW,3HIRL,6HIMPULS,1HE/
DATA ST1,ST2/1H ,1H ,1H ,6H WITH ,6HEXIT V,6HANES /
PI= 3.1416
98 WRITE(6,100)

```

```

100 FORMAT(1H1,49X,33HTURBINE VELOCITY DIAGRAM ANALYSIS)
    READ(5,99) TITLE
    99 FORMAT(13A6)
    WRITE (6,101) TITLE
101 FORMAT(1H ,13A6)
    ITIT=0
    1 READ(5,INPUT)
    GO TO (95,96),IU
    95 J=1.
    G=1.
    CCN=2.*PI
    CCP=1000.
    GO TO 97
    96 J=778.
    G=3.*.17
    CCN=60.
    CCP=550.
    97 R1=.9
    ES=.8
    ALP= ALPHA
    REX=RREX
    XREX=REX
    IF (IALPH.EQ.1) ALP=0.0
    IF (IALPH.EQ.0) XREX=0.0
    IF (IPR.EQ.0) PRTS=0.0
    IF (IPR.EQ.1) POW=0.0
    DO 14 I=1,3
    ST(I)=ST1(I)
14 IF (IEV.EQ.1) ST(I)=ST2(I)
    GO TO (13,15,17),IDIAM
13 DX=DH
    DMEX=(1.+REX)/2.*DEX/REX
    DMIN=(1.+R1)/2.*DIN/R1
    GO TO 19
15 DX=DM
    DMEX=DEX
    DMIN=DIN
    GO TO 19
17 DX=DT
    DMEX=(1.+REX)/2.*DEX
    DMIN=(1.+R1)/2.*DIN
19 WRITE(6,109)
109 FORMAT(1H )
    WRITE(6,110)DX,DX,POW,W,TTIN,PTIN,RPM,DIN,DEX,XREX,ALP,R,GAM,MU,
    1KLOSS,E,PRTS
110 FORMAT(126H0      SHAFT      MASS      INLET      INLET      ROTATIVE      INLET
1  EXIT      EXIT      STATOR      GAS      HEAT      GAS      TURBINE      AXIAL
1  T-S/
2  POWER      FLOW      TEMP      PRESS      SPEED      ,A2,6H DIA ,A2,
2
3S  EX ANG CONST      CAPAC      VISCOSITY      LOSS      VcL SQ PRESS/
465X,5HRATIO,19X,5HRATIO,14X,19HCOEF      RATIO      RATIO/
5
6E11.3,F7.3,2F8.3)
    ALPH1= ALPHA*.017453
    X=GAM/(GAM-1.)
    CP= X*R/J
    IF (IPR.EQ.1) DHID=CP*TTIN*(1.-(1./PRTS)**(1./X))
    IF (IPR.EQ.1) PEX=PTIN/PRTS

```

```

IF(IPR.EQ.1) GO TO 53
DELHT=CCP*POW/W/J
TTEX= TTIN-DELHT/CP
IF(TTEX.LT.0.0) GO TO 20
53 N=NMIN
CONV=.01
37 IF(IU.EQ.1) DN=DMEX/100.
IF(IU.EQ.2) DN=DMEX/12.
IF(IU.EQ.1) D1=DMIN/100.
IF(IU.EQ.2) D1=DMIN/12.
11 U(1)=PI*RPM*D1/CCN
U(N)=PI*RPM*DN/CCN
NN= FLOAT(N)
IF(N.EQ.1) SUMUSQ=U(N)*U(N)
IF(N.EQ.1) GO TO 3
SUMUSQ= 0.0
DO I=1,N
U(I)= (U(N)-U(1))/(NN-1.)*(FLOAT(I)-1.) + U(1)
UISQ=J(I)**2
2 SUMUSQ=SUMUSQ+UISQ
3 IF(IPR.EQ.0) GO TO 55
54 DELHT=ES*DHID
TTEX=TTIN-DELHT/CP
ESA=ES
55 LAM=SUMUSQ/G/J/DELHT
GO TO (61,62,63,64), IVD
61 Q=(LAM+1.)/2.
Q2=(LAM-1.)/2.
FLS=2.-LAM
VX1=VD1
VX2=VD2
VX3=VD3
GO TO 65
62 Q=1.0
Q2=.0
FLS=1.0
VX1=VD4
VX2=VD5
VX3=VD6
GO TO 65
63 Q=LAM+.5
Q2=LAM-.5
FLS=1.0
IF(LAM.LT..5) FLS=2.*(1.-LAM)
VX1=VD7
VX2=VD8
VX3=VD3
GO TO 65
64 IF(LAM.GE..5) GO TO 62
GO TO 63
65 DVUN=J(N)/LAM
VU1N= Q*DVUN
VU2N=Q2*DVUN
4 COT= COTAN(ALPH1)
C1=(1.+2.*COT**2)*Q*Q
C1= C1+(Q-1.)**2
D=2.*COT**2*Q*Q+(Q-LAM)**2+(Q-LAM-1.)**2
RE= W/MU/D1*2.
IF(N.EQ.1) RE=W/MU/DN*2.

```

```

A=KLOSS/COT/RE**.2
A1=A*(CI+2.*D)
AI=A*(FLS*CI+2.*D)
B=E*COT**2*Q*Q+Q2**2
ET1=LAM/(LAM+A1/2.)
ESI=LAM/(LAM+(A1+B)/2.)
ETI=LAM/(LAM+AI/2.)
ESI=LAM/(LAM+(AI+B)/2.)
IF (IEV.EQ.1) GO TO 34
IF (N-1) 5,5,6
5 ES= ESI
GO TO 7
6 ES=1./((U(1)*U(1)/ET1/SUMUSQ+(1.-(U(1)*U(1)+U(N)*U(N))/SUMUSQ)/ETI
1+U(N)*U(N)/ESI/SUMUSQ)
GO TO 7
34 CL=2.*COT**2*Q*Q+Q2**2
FLSL=1.
A1L=A1+A*CL*FLSL
AIL=AI+A*CL*FLSL
BL=E*COT**2*Q*Q
ET1L=LAM/(LAM+A1L/2.)
ES1L=LAM/(LAM+(A1L+BL)/2.)
ETIL=LAM/(LAM+AIL/2.)
ESIL=LAM/(LAM+(AIL+BL)/2.)
IF (N-1) 35,35,36
35 ES=ES1L
GO TO 7
36 ES=1./((U(1)*U(1)/ET1/SUMUSQ+(1.-(U(1)*U(1)+U(N)*U(N))/SUMUSQ)/ETI
1+U(N)*U(N)/ESIL/SUMUSQ)
7 IF (IPR) 56,56,57
56 ZZ=DELHT/CP/TTIN/ES
IF (ZZ.GE.1.0) GO TO 21
PEX=PTIN*(1.-ZZ)**X
GO TO 58
57 IF (ABS (ES-ESA).LT..0001) GO TO 58
ES=(ES+ESA)/2.
GO TO 54
58 VXND=Q*COT*DVUN
VXN=VXND*SQRT(E)
V2N= SQRT(VU2N**2+VXN**2)
V2NR=V2N
IF (IEV.EQ.1) V2N=VXN
TEX= TTEX-V2N**2/2./G/J/CP
IF (TEX.LE.0.0) GO TO 22
IF (IU.EQ.1) RHOEX=PEX/R*.0000./TEX
IF (IU.EQ.2) RHOEX=PEX/R*.44./TEX
IF (IALPH.EQ.1) GO TO 8
AEX= W/RHOEX/VXN
AA= AEX/PI/UN**2
IF (AA.GE.1.0) GO TO 23
REX1=REX
REX= (1.-AA)/(1.+AA)
IF (IDIAM.EQ.2) GO TO 9
IF (N.EQ.1) CONV=.0001
IF (ABS (REX-REX1).LT.CONV) GO TO 9
IF (IDIAM.EQ.3) GO TO 33
DMEX=(1.+REX)/2.*DEX/REX
GO TO 37

```

```

35 DMEX=(1.+REX)/2.*DEX
GO TO 37
8 AEX= PI*DN**2*(1.-REX)/(1.+REX)
31 VXN= W/AEX/RHOEX
V2N=SQRT(VU2N**2+VXN**2)
V2NR=V2N
IF(IEV.EQ.1) V2N=VXN
TEX=TTEX-V2N**2/2./G/J/CP
RHOUX1=RHOEX
IF(IU.EQ.1) RHOEX=PEX/R*10000./TEX
IF(IU.EQ.2) RHOEX=PEX/R*144./TEX
IF(ABS(RHOEX-RHOEX1).GT..001*RHOEX) GO TO 31
ALPH= ALPH1
VXND=VXN/SQRT(E)
ALPH1=ATAN2(VU1N,VXND)
IF(ABS(ALPH1-ALPH).GT..0.02) GO TO 4
9 ALPH2=ATAN2(VU2N,VXND)
PTEX=PEX*(TTEX/TEX)**X
DHTID=CP*TTIN*(1.-(PTEX/PTIN)**(1./X))
ET=DELHT/DHTID
POW=DELHT*W/CCP*J
WU1N=VU1N-U(N)
WU2N=VU2N-U(N)
BET1=ATAN2(WU1N,VXND)
BET2=ATAN2(WU2N,VXND)
W2N=SQRT(WU2N**2+VXN**2)
W1N=SQRT(WU1N**2+VXND**2)
V1N=SQRT(VU1N**2+VXND**2)
TTRN=TTEX-(V2NR**2-W2N**2)/2./G/J/CP
TT1N=TTRN-(W1N**2-V1N**2)/2./G/J/CP
WC=SQRT(2.*GAM/(GAM+1.)*G*R)
VCR2N=WC*SQRT(TTEX)
WCRN=WC*SQRT(TTRN)
VCR1N=WC*SQRT(TT1N)
V1OVCR=V1N/VCR1N
W1OVCR=W1N/WCRN
W2OVCR=W2N/WCRN
V2OVCR=V2NR/VCR2N
ASEX= SQRT(GAM*G*R*TEX)
MXEX= VXN/ASEX
DTEX= DMEX*2./((1.+REX)
DHEX= REX*DTEX
IF(V.EQ.1) GO TO 51
DELH1= U(1)**2/G/J/LAM
TT21= TTIN-DELH1/CP
PT21= PTIN*(1.-DELH1/CP/TTIN/ET1)**X
DVU1 = U(1)/LAM
VX1=Q*COT*DVU1*SQRT(E)
VU21= Q2*DVU1
VU11= Q*DVU1
V21= SQRT(VU21**2+VX1**2)
T21= TT21-V21**2/2./G/J/CP
P21= PT21*(T21/TT21)**X
IF(IU.EQ.1) RHO21=P21/R*10000./T21
IF(IU.EQ.2) RHO21=P21/R*144./T21
A21=W/RHO21/VX1
AA1=A21/PI/D1/D1
IF(AA1.GE.1.0) GO TO 24
R11=R1

```

```

R1= (1.-AA1)/(1.+AA1)
IF(IDIAM.EQ.2) GO TO 49
IF(ABS(R1-R11).LT.CONV) GO TO 41
44 IF(IDIAM=2) 45,49,47
45 DMI=(1.+R1)/2.*DIN/R1
   DMEX=(1.+REX)/2.*DEX/REX
   GO TO 37
47 DMIN=(1.+R1)/2.*DIN
   DMEX=(1.+REX)/2.*DEX
   GO TO 37
41 IF(CONV-.001) 42,42,43
42 GO TO 49
43 CONV=.0001
   GO TO 44
49 DT1=DMIN*2./(1.+R1)
   DH1 = R1*DT1
   GO TO 52
51 R1=REX
   DT1=DTEX
   DH1=DHEX
52 ALPHA1=ALPH1/.017453
   ALPHA2= ALPH2/.017453
   BETA1=BET1/.017453
   BETA2=BET2/.017453
   PRT= PTIN/PTEX
   PRS= PTIN/PEX
   WRITE(6,120)N,ST,LAM,VX11,VX2,VX3,DTEX,TTEX,ALPHA1,U(1),DHEX,TEX,
1ALPHA2,U(N),REX,PTEX,BETA1,VU1N,DT1,PEX,BETA2,VU2N,DH1,PRT,ET,VXN,
2R1,PRS,ES,MXEX
120 FORMAT(8HSTAGES=,I2,3A6,2X,27HSTAGE SPEED-WORK PARAMETER=,F5.3,
126X,13HDIAGRAMS ARE ,3A6
1 /20H EXIT TIP DIAMETER =,F6.2,4X,18HEXIT TOTAL
1 TEMP =,F7.2,5X,18HSTATOR EXIT ANGLE=,F6.2,4X,23HFIRST STAGE MEAN
2 SPEED=,F8.2/20H EXIT HUB DIAMETER =,F6.2,4X,18HEXIT STATIC TEMP =
3,F7.2,5X,18HSTAGE EXIT ANGLE =,F6.2,4X,23HLAST STAGE MEAN SPEED =,
4F8.2/20H EXIT RADIUS RATIO =,F6.4,4X,18HEXIT TOTAL PRESS =,F7.2,5X
5,18HROTOR INLET ANGLE=,F6.2,4X,23HLAST STAGE INLET SWIRL=,F8.2/20H
6 INLET TIP DIAMETER=,F6.2,4X,18HEXIT STATIC PRESS=,F7.2,5X,18HROTO
7R EXIT ANGLE =,F6.2,4X,23HLAST STAGE EXIT SWIRL =,F8.2/20H INLET H
8UB DIAMETER=,F6.2,4X,18HT-T PRESS RATIO =,F7.3,5X,18HTOTAL EFFICI
9ENCY =,F5.3,5X,23HLAST STAGE AXIAL VELOC=,F8.2/20H INLET RADIUS RA
1TIO=,F6.4,4X,18HT-S PRESS RATIO =,F7.3,5X,18HSTATIC EFFICIENCY=,
2F5.3,5X,23HEXIT AXIAL MACH NUMBER=,F7.4)
   IF(IPR.EQ.1.AND.IU.EQ.1) DELHT=DELHT/1000.
   IF(IPR.EQ.1) WRITE(6,121) POW,DELHT
121 FORMAT(17H SHAFT POWER =,F9.1,4X,18HSPECIFIC WORK =,F7.2)
   WRITE(6,122)V1OVCR,W1OWCR,W2OWCR,V2OVCR
122 FORMAT(20H LAST STG (V1/VCR1)=,F6.4,4X,18HLAST STG(W1/WCR1)=,F7.4,
15X,.8HLAST STG(W2/WCR2)=,F6.4,4X,23HLAST STG (V2/VCR2) =,F7.4)
12 N=N+1
   CONV=.01
   IF(N.LE.NMAX) GO TO 11
   IF(ITIT.EQ.1) GO TO 98
   GO TO 1
20 WRITE(6,130)
130 FORMAT(1H0,5X,19HINSUFFICIENT ENERGY)
   GO TO 1
21 WRITE(6,140)N

```

```
140 FORMAT(1H0,7HSTAGES=,I2,5X,25HINSUFFICIENT IDEAL ENERGY)
    GO TO 12
    22 WRITE(6,150)N
150 FORMAT(1H0,7HSTAGES=,I2,5X,12HNEGATIVE TEX)
    GO TO 12
    23 WRITE(6,160)N
160 FORMAT(1H0,7HSTAGES=,I2,5X,22HINSUFFICIENT EXIT AREA)
    R1=.9
    REX=.7
    GO TO 12
    24 WRITE(6,170) N
170 FORMAT(1H0,7HSTAGES=,I2,5X,23HINSUFFICIENT INLET AREA)
    R1=.9
    REX=.7
    GO TO 12
    END
```

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 16, 1971,
764-74.

REFERENCE

1. Stewart, Warner L.: A Study of Axial-Flow Turbine Efficiency Characteristics in Terms of Velocity Diagram Parameters. Paper 61-WA-37, ASME, 1961.

TABLE I. - INPUT FORM WITH SAMPLE DATA

LINE NUMBER	FORTRAN STATEMENT	IDENTIFICATION
	THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNITS ARE USED.	
	\$ INPUT PTIN=113.1,TTIN=2660.,MU=.376E-04,R=53.37,GAM=1.302,DIN=22.,DEX=24.,	
	RPM=11400.,POW=12900.,W=53.5,ALPHA=65.,KLOSS=.35,NMIN=1,NMAX=2,E=1.2,	
	IALPH=0,IDIAM=2,IVD=1,IEV=0,IPR=0,IU=2 \$	
	\$ INPUT IDIAM=1,RREX=.8,DIN=20.,DEX=21.,ITIT=1 \$	
	THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA	
	\$ INPUT IALPH=1,IEV=1,NMIN=2 \$	
	\$ INPUT IALPH=0,IEV=0,IVD=2 \$	
	\$ INPUT IVD=3 \$	
	\$ INPUT IPR=1,IVD=1,PRTS=3.66 \$	

MASSACHUSETTS 9-14-59

TABLE II. - OUTPUT FORM WITH SAMPLE DATA

THESE ARE SAMPLE CASES FOR THIS PROGRAM. U.S. CUSTOMARY UNITS ARE USED.

TURBINE VELOCITY DIAGRAM ANALYSIS

12900.0 53.50 2660.00 113.10 11400.00 22.00 24.00 0. 65.00 53.37 1.302 0.376E-04 0.350 1.200 0.

STAGES= 1
 EXIT TIP DIAMETER = 26.75
 EXIT HUB DIAMETER = 21.25
 EXIT RADIUS RATIO = 0.7942
 INLET TIP DIAMETER = 26.75
 INLET HUB DIAMETER = 21.25
 INLET RADIUS RATIO = 0.7942
 LAST STG (V1/VCR1) = 1.1572

STAGE SPEED-WORK PARAMETER=0.334
 EXIT TOTAL TEMP = 2083.64
 EXIT STATIC TEMP = 1887.83
 EXIT TOTAL PRESS = 32.64
 EXIT STATIC PRESS = 21.33
 T-T PRESS RATIO = 3.465
 T-S PRESS RATIO = 5.303
 LAST STG(W1/WCR1) = 0.7587

STAGE SPEED-WORK PARAMETER=0.615
 EXIT TOTAL TEMP = 2083.64
 EXIT STATIC TEMP = 2030.85
 EXIT TOTAL PRESS = 34.44
 EXIT STATIC PRESS = 30.84
 T-T PRESS RATIO = 3.284
 T-S PRESS RATIO = 3.668
 LAST STG(W1/WCR1) = 0.3937

STAGES= 2
 EXIT TIP DIAMETER = 27.11
 EXIT HUB DIAMETER = 20.89
 EXIT RADIUS RATIO = 0.7703
 INLET TIP DIAMETER = 24.09
 INLET HUB DIAMETER = 19.91
 INLET RADIUS RATIO = 0.8268
 LAST STG (V1/VCR1) = 0.8018

12900.0 53.50 2660.00 113.10 11400.00 20.00 21.00 0. 65.00 53.37 1.302 0.376E-04 0.350 1.200 0.

STAGE SPEED-WORK PARAMETER=0.328
 EXIT TOTAL TEMP = 2083.64
 EXIT STATIC TEMP = 1883.59
 EXIT TOTAL PRESS = 32.56
 EXIT STATIC PRESS = 21.07
 T-T PRESS RATIO = 3.473
 T-S PRESS RATIO = 5.367
 LAST STG(W1/WCR1) = 0.7673

STAGE SPEED-WORK PARAMETER=0.620
 EXIT TOTAL TEMP = 2083.64
 EXIT STATIC TEMP = 2031.21
 EXIT TOTAL PRESS = 34.46
 EXIT STATIC PRESS = 30.87
 T-T PRESS RATIO = 3.282
 T-S PRESS RATIO = 3.663
 LAST STG(W1/WCR1) = 0.3922

STAGES= 1
 EXIT TIP DIAMETER = 26.58
 EXIT HUB DIAMETER = 21.00
 EXIT RADIUS RATIO = 0.7900
 INLET TIP DIAMETER = 26.58
 INLET HUB DIAMETER = 21.00
 INLET RADIUS RATIO = 0.7900
 LAST STG (V1/VCR1) = 1.1622

STAGES= 2
 EXIT TIP DIAMETER = 27.20
 EXIT HUB DIAMETER = 21.00
 EXIT RADIUS RATIO = 0.7721
 INLET TIP DIAMETER = 24.16
 INLET HUB DIAMETER = 20.00
 INLET RADIUS RATIO = 0.8279
 LAST STG (V1/VCR1) = 0.8012

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1193.81
 LAST STAGE MEAN SPEED = 1193.81
 LAST STAGE INLET SWIRL = 2383.74
 LAST STAGE EXIT SWIRL = -1189.93
 LAST STAGE AXIAL VELOC = 1217.71
 EXIT AXIAL MACH NUMBER = 0.5928
 LAST STG (V2/VCR2) = 0.8464

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1094.32
 LAST STAGE MEAN SPEED = 1193.81
 LAST STAGE INLET SWIRL = 1567.87
 LAST STAGE EXIT SWIRL = -374.06
 LAST STAGE AXIAL VELOC = 800.93
 EXIT AXIAL MACH NUMBER = 0.3755
 LAST STG (V2/VCR2) = 0.4394

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1098.50
 LAST STAGE MEAN SPEED = 1198.77
 LAST STAGE INLET SWIRL = 1566.79
 LAST STAGE EXIT SWIRL = -368.02
 LAST STAGE AXIAL VELOC = 800.28
 EXIT AXIAL MACH NUMBER = 0.3756
 LAST STG (V2/VCR2) = 0.4379

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1183.50
 LAST STAGE MEAN SPEED = 2394.15
 LAST STAGE EXIT SWIRL = -1210.65
 LAST STAGE AXIAL VELOC = 1223.63
 EXIT AXIAL MACH NUMBER = 0.5960
 LAST STG (V2/VCR2) = 0.8555

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1098.50
 LAST STAGE MEAN SPEED = 1198.77
 LAST STAGE INLET SWIRL = 1566.79
 LAST STAGE EXIT SWIRL = -368.02
 LAST STAGE AXIAL VELOC = 800.28
 EXIT AXIAL MACH NUMBER = 0.3756
 LAST STG (V2/VCR2) = 0.4379

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1183.50
 LAST STAGE MEAN SPEED = 2394.15
 LAST STAGE EXIT SWIRL = -1210.65
 LAST STAGE AXIAL VELOC = 1223.63
 EXIT AXIAL MACH NUMBER = 0.5960
 LAST STG (V2/VCR2) = 0.8555

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1098.50
 LAST STAGE MEAN SPEED = 1198.77
 LAST STAGE INLET SWIRL = 1566.79
 LAST STAGE EXIT SWIRL = -368.02
 LAST STAGE AXIAL VELOC = 800.28
 EXIT AXIAL MACH NUMBER = 0.3756
 LAST STG (V2/VCR2) = 0.4379

DIAGRAMS ARE SYMMETRICAL
 FIRST STAGE MEAN SPEED = 1094.32
 LAST STAGE MEAN SPEED = 1193.81
 LAST STAGE INLET SWIRL = 1567.87
 LAST STAGE EXIT SWIRL = -374.06
 LAST STAGE AXIAL VELOC = 800.93
 EXIT AXIAL MACH NUMBER = 0.3755
 LAST STG (V2/VCR2) = 0.4394

TABLE II. - Concluded. OUTPUT FORM WITH SAMPLE DATA

TURBINE VELOCITY DIAGRAM ANALYSIS															
THIS IS AN ADDITIONAL TITLE CALLED FOR BY ITIT IN PREVIOUS CASE DATA															
SHAFT POWER	MASS FLOW	INLET TEMP	INLET PRESS	ROTATIVE SPEED	INLET HB DIA	INLET HB DIA	EXIT HB DIA	EXIT RADIUS	STATOR EX ANG	GAS CONST	HEAT CAPAC RATIO	GAS VISCOSITY	TURBINE LOSS CCEF	AXIAL VEL SQ RATIO	T-S PRESS RATIO
12900.0	53.50	2660.00	113.10	11400.00	20.00	21.00	21.00	0.8000	0.	53.37	1.302	0.376E-04	0.350	1.200	0.
STAGES= 2 WITH EXIT VANES															
STAGE SPEED-WORK PARAMETER=0.598															
EXIT TIP DIAMETER = 26.25															
EXIT HUB DIAMETER = 21.00															
EXIT STATIC TEMP = 2017.58															
EXIT TOTAL PRESS = 34.36															
EXIT TIP DIAMETER = 23.50															
EXIT STATIC PRESS = 29.90															
INLET HUB DIAMETER = 20.00															
INLET STATIC PRESS = 3.292															
INLET TIP DIAMETER = 0.8510															
INLET PRESS RATIO = 3.782															
LAST STG (V1/VCR1) = 0.8396															
LAST STG (V2/VCR2) = 0.4724															
DIAGRAMS ARE SYMMETRICAL															
FIRST STAGE MEAN SPEED = 1081.87															
LAST STAGE MEAN SPEED = 1175.15															
LAST STAGE INLET SWIRL = 1570.07															
LAST STAGE EXIT SWIRL = -354.52															
LAST STAGE AXIAL VELOC = 988.85															
EXIT AXIAL MACH NUMBER = 0.4856															
LAST STG (V2/VCR2) = 0.5293															
STAGES= 2															
STAGE SPEED-WORK PARAMETER=0.598															
EXIT TIP DIAMETER = 26.25															
EXIT HUB DIAMETER = 21.00															
EXIT STATIC TEMP = 2015.45															
EXIT TOTAL PRESS = 33.91															
INLET TIP DIAMETER = 23.46															
INLET STATIC PRESS = 29.38															
INLET HUB DIAMETER = 20.00															
INLET PRESS RATIO = 3.336															
INLET TIP DIAMETER = 0.8526															
INLET PRESS RATIO = 3.850															
LAST STG (V1/VCR1) = 1.0060															
LAST STG (V1/VCR1) = 0.5892															
DIAGRAMS ARE ZERO EXIT SWIRL															
FIRST STAGE MEAN SPEED = 1080.83															
LAST STAGE MEAN SPEED = 1175.27															
LAST STAGE INLET SWIRL = 1966.71															
LAST STAGE EXIT SWIRL = 0.															
LAST STAGE AXIAL VELOC = 1004.68															
EXIT AXIAL MACH NUMBER = 0.4733															
LAST STG (V2/VCR2) = 0.4994															
STAGES= 2															
STAGE SPEED-WORK PARAMETER=0.592															
EXIT TIP DIAMETER = 26.05															
EXIT HUB DIAMETER = 21.00															
EXIT STATIC TEMP = 1999.29															
EXIT TOTAL PRESS = 33.17															
INLET TIP DIAMETER = 23.25															
INLET STATIC PRESS = 27.76															
INLET HUB DIAMETER = 20.00															
INLET PRESS RATIO = 3.410															
INLET TIP DIAMETER = 0.8601															
INLET PRESS RATIO = 4.074															
LAST STG (V1/VCR1) = 1.1039															
LAST STG (V1/VCR1) = 0.6906															
DIAGRAMS ARE IMPULSE															
FIRST STAGE MEAN SPEED = 1075.74															
LAST STAGE MEAN SPEED = 1170.14															
LAST STAGE INLET SWIRL = 2156.13															
LAST STAGE EXIT SWIRL = 162.16															
LAST STAGE AXIAL VELOC = 1102.46															
EXIT AXIAL MACH NUMBER = 0.5215															
LAST STG (V2/VCR2) = 0.5555															
STAGES= 2															
STAGE SPEED-WORK PARAMETER=0.620															
EXIT TIP DIAMETER = 27.20															
EXIT HUB DIAMETER = 21.00															
EXIT STATIC TEMP = 2031.00															
EXIT TOTAL PRESS = 34.48															
INLET TIP DIAMETER = 24.16															
INLET STATIC PRESS = 30.90															
INLET HUB DIAMETER = 20.00															
INLET PRESS RATIO = 3.280															
INLET TIP DIAMETER = 0.8278															
INLET PRESS RATIO = 12892.7															
SHAFT POWER = 12892.7															
LAST STG (V1/VCR1) = 0.8009															
LAST STG (V1/VCR1) = 0.3920															
DIAGRAMS ARE SYMMETRICAL															
FIRST STAGE MEAN SPEED = 1098.30															
LAST STAGE MEAN SPEED = 1198.70															
LAST STAGE INLET SWIRL = 1566.21															
LAST STAGE EXIT SWIRL = -367.51															
LAST STAGE AXIAL VELOC = 800.08															
EXIT AXIAL MACH NUMBER = 0.3754															
LAST STG (V2/VCR2) = 0.4376															

Speed-work parameter	Diagram type		
	Zero exit swirl	Impulse	Symmetrical
0.25			
0.5			
1.0			

Figure 1. - Effect of diagram type and speed-work parameter on velocity diagram shape.

|